# Mathematics 

## Paper 2 (Calculator)

## Higher Tier

Churchill Paper 2E - Marking Guide

Method marks (M) are awarded for a correct method which could lead to a correct answer
Accuracy marks (A) are awarded for a correct answer, having used a correct method, although this can be implied
(B) marks are awarded independent of method

Churchill
Maths
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1 (a) $180-2 \times 31=180-62=118^{\circ}$
$a=360-118=242^{\circ}$

(b) External angle of regular octagon $=360 \div 8=45^{\circ}$

External angle of regular hexagon $=360 \div 6=60^{\circ}$
$b=45+60=105^{\circ}$
$95^{\circ}$

$115^{\circ}$
B1 Total 2
$2 \quad\left(2 p^{3}\right)^{2}=2^{2} \times\left(p^{3}\right)^{2}=4 p^{6}$
$2 p^{6} \quad 4 p^{5} \quad 4 p^{6} \quad 32 p^{5} \quad$ B1 Total 1
$3 \sqrt{0.58}=0.7615 \ldots$
$\sqrt{0.59}=0.7681$.
$\frac{10}{13}=0.7692 \ldots$
$\frac{13}{17}=0.7647 \ldots$
$\frac{17}{22}=0.7727 \ldots$
$\frac{35}{46}=0.7608 \ldots$
$\frac{10}{13} \quad \frac{13}{17} \quad \frac{17}{22} \quad \frac{35}{46}$
B1 Total 1

4 Kevin's share $=3 \times 36=£ 108$ M1
8 portions $=108$ so 1 portion $=108 \div 8=£ 13.50$
M1
Jamal gets 5 portions $=5 \times 13.50=£ 67.50$
A1
Total 3

5 (a) $\approx 300-260=40$
(b)

$\approx 12$ teachers (from line of best fit, nearest whole number)
(c) $\quad \frac{25-5}{445-50}=0.051(2 \mathrm{dp})$
(d) e.g. You need an extra 0.051 teachers for every extra pupil
[OR: You need an extra teacher for every extra 20 pupils]
Total 6

6 e.g. $20 \%$ off is $\frac{1}{5}$ off
Buy 2 get 1 half price means he would pay $2 \frac{1}{2}$ times the
usual price for 3 packets
The fraction of full price he pays is $\frac{2 \frac{1}{2}}{3}=\frac{5}{6}$
The discount is $\frac{1}{6}$ M1
$\frac{1}{5}$ is larger than $\frac{1}{6}$ so $20 \%$ off is better value A1
[Can get full marks with an assumed price and suitable words]

Total 3

7
(a)

(b) $(3,3)$ and $(4,4)$

B1
(c) Rotation by $90^{\circ}$ clockwise about the point $(0,0)$

8 A kite has two pairs of equal sides
Here:

$$
\begin{array}{ll}
2 p+2=3 p-3 & \text { M1 } \\
2 p+5=3 p & \\
5=p & \text { A1 }
\end{array}
$$

So, $2 p+2=2 \times 5+2=12$
Each half of kite is a right-angled triangle
Base $=2 p+2=12 \mathrm{~cm}$ and perpendicular height $=p=5 \mathrm{~cm}$
Area of half of kite $=\frac{1}{2} \times 5 \times 12=30 \mathrm{~cm}^{2} \quad$ M1
Area of kite $=2 \times 30=60 \mathrm{~cm}^{2} \quad$ A1
Total 4

9 (a) The mean
B1
e.g. There is no prize of 50 p so that cannot be the mode.

With 5 values, the median will be the $3^{\text {rd }}$ value (in order) and as there is no prize of 50 p the median cannot be 50 p.
(b) e.g. He has assumed the ball is equally likely to go through each of the gates.
(c) e.g. To go through the outer gates the ball has to move quite a bit to the side. Hence the ball is less likely to go through the outer gates and his assumption is not reasonable. The outer gates win the bigger prizes so the true mean prize will be considerably less (and almost certainly less than the 40 p it costs for a roll!)

10
(a) $=x^{2}-x-5 x+5$
$=x^{2}-6 x+5$
$x^{2}-6 x+5 \quad x^{2}-6 x-5 \quad x^{2}-4 x+5 \quad x^{2}-5 x+6$
(b) $(x-1)(x+10) \quad(x+2)(x-5)$
$(x-2)(x-5) \quad(x-2)(x+5)$
B1 Total 2

11
(a) $\approx-3.8 \quad$ B1
(b) $x \approx-1, x \approx 4$

B1
Total 2
$12 y=\frac{1}{3}(x-2)$
For inverse, swap $x$ and $y: \quad x=\frac{1}{3}(y-2)$
$3 x=y-2$
$3 x+2=y$
$\frac{3}{x+2} \quad 3(x+2) \quad \frac{3}{2-x} \quad 3 x+2$
B1 Total 1
$13 \sqrt[4]{40000}=\sqrt[4]{4} \times \sqrt[4]{10000}=\sqrt[4]{4} \times 10 \quad$ M1
$\sqrt[4]{4}=\sqrt{(\sqrt{4})}=\sqrt{2}$
M1
$\sqrt[4]{40000}=\sqrt[4]{4} \times 10=10 \sqrt{2} \quad$ A1
Total 3

14 (a) Gradient of graph $\approx \frac{60-0}{200-0}=0.3$
So, height increases at a rate of 0.3 cm per second volume increases at a rate of $0.3 \times 40 \times 50$ $=600 \mathrm{~cm}^{3}$ per second M1
water is supplied at a rate of 600 ml per second

$$
\begin{aligned}
& =600 \times 3600 \mathrm{ml} \text { per hour } \\
& =2160000 \mathrm{ml} \text { per hour } \\
& =2160 \text { litres per hour } \quad \text { A1 }
\end{aligned}
$$

(b) e.g. I have assumed that the sides of the tank are so thin that they can be ignored.

B1
As they will have a thickness, the volume supplied is less so the rate at which the water is supplied will be less meaning that my answer is too big.

15 Equate

$$
\begin{array}{ll}
3 x^{2}-x+7=9-6 x & \text { M1 } \\
3 x^{2}+5 x-2=0 & \text { M1 } \\
(3 x-1)(x+2)=0 & \text { A1 }
\end{array}
$$

When $x=\frac{1}{3}, y=7$

When $x=-2, y=21$
A1
So $x=-2, y=21$ or $x=\frac{1}{3}, y=7$
Total 4

16 (a) e.g. As the density of gold is higher, a certain volume of gold will contribute more to the total mass than the same volume of silver. The $\%$ gold will be higher by mass than by volume.
(b) Let volume of gold be $\mathrm{xcm}{ }^{3}$

Mass $=$ density $\times$ volume
Mass of gold $=19.3 x$ grams
Volume of silver $=(0.6-x) \mathrm{cm}^{3}$
Mass of silver $=10.5(0.6-x)$ grams
Total mass $=10$ grams so:

$$
\begin{aligned}
& 19.3 x+10.5(0.6-x)=10 \\
& 19.3 x+10.5 \times 0.6-10.5 x=10 \\
& 19.3 x-10.5 x=10-6.3 \\
& 8.8 x=3.7 \\
& x=\frac{3.7}{8.8}=0.4204 \ldots
\end{aligned}
$$

Percentage of volume that is gold $=\frac{0.4204 . .}{0.6} \times 100 \%$

$$
=70.1 \% \text { (3sf) }
$$

Hence about $70 \%$ of the volume is gold

M1
A1
Total 5

17 (a) Area $=\frac{1}{2} \times 6 \times 8 \times \sin 30^{\circ}$

$$
=\frac{1}{2} \times 48 \times \frac{1}{2}
$$

$=12 \mathrm{~cm}^{2}$
$12 \mathrm{~cm}^{2} 12 \sqrt{2} \mathrm{~cm}^{2} \quad 12 \sqrt{3} \mathrm{~cm}^{2} \quad 24 \mathrm{~cm}^{2}$
e.g. If angle $P Q R$ is a right angle then $\cos 45^{\circ}=\frac{3 \sqrt{2}}{6}$
We know $\quad \cos 45^{\circ}=\frac{1}{\sqrt{2}} \quad$ B1
And we have $\quad \frac{3 \sqrt{2}}{6}=\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{(\sqrt{2})^{2}}=\frac{1}{\sqrt{2}}=\cos 45^{\circ} \quad \mathrm{M} 1$
Therefore angle $P Q R$ must be a right angle so triangle $P Q R$ is a right-angled triangle and Sam is correct

A1
Total 4

18 Number of games won out of 523 must be less than $52.5 \%$
$0.525 \times 523=274.575$
Must be whole number so greatest $=274$ A1
Number of games won out of 543 must be at least $52.5 \%$
$0.525 \times 543=285.075$
Must be whole number so least $=286$
Smallest number won $=286-274=12$ games
M1 A1 Total 4

19 Volume of cake $=\frac{1}{3} \times \pi \times 8^{2} \times 20=\frac{1280}{3} \pi$
Volume of one piece $=\frac{1}{2} \times \frac{1280}{3} \pi=\frac{640}{3} \pi$ M1
Let base radius of top piece be $r$ and height be $h$
By similar triangles $\quad \frac{r}{h}=\frac{8}{20}$ M1

Hence

$$
r=\frac{2}{5} h
$$

Volume of top piece $=\frac{1}{3} \times \pi \times\left(\frac{2}{5} h\right)^{2} \times h=\frac{640}{3} \pi$

$$
\frac{4}{25} h^{3}=640
$$

$$
h^{3}=\frac{25}{4} \times 640=4000
$$

$$
h=\sqrt[3]{4000}=15.874 \ldots
$$

Height above base $=20-15.874=4.125 \ldots=4.1 \mathrm{~cm}(1 \mathrm{dp})$
[Quick method: From whole to top half, volume scale factor $=0.5$
so length scale factor $=\sqrt[3]{0.5}=0.7937 \ldots$
so height above base $=20-0.7937 \times 20=4.1]$
$20 \quad \mathrm{P}\left(1^{\text {st }}\right.$ is red $)=\frac{x}{x+2 x}=\frac{x}{3 x}=\frac{1}{3}$
If $1^{\text {st }}$ is red, there are now $(x+4)$ red and $x$ blue in box Q
$\mathrm{P}\left(2^{\text {nd }}\right.$ is red given $1^{\text {st }}$ is red $)=\frac{x+4}{2 x+4}$
M1
$\mathrm{P}($ both are red $)=\frac{1}{3} \times \frac{x+4}{2 x+4}=\frac{1}{4}$
$4(x+4)=3(2 x+4)$
$4 x+16=6 x+12$
$4=2 x$
$x=2$
So $P$ has 2 red and 4 blue and $Q$ has 5 red and 2 blue
$\mathrm{P}\left(1^{\text {st }}\right.$ is blue $)=\frac{4}{6}=\frac{2}{3}$
If $1^{\text {st }}$ is blue there are now 5 red and 3 blue in box $Q$
$P\left(2^{\text {nd }}\right.$ is blue given $1^{\text {st }}$ is blue $)=\frac{3}{8}$ M1
$P($ both are blue $)=\frac{2}{3} \times \frac{3}{8}=\frac{2}{8}=\frac{1}{4}$

21 Let short side of rectangle be $x \mathrm{~cm}$
So side of equilateral triangle $=2 x \mathrm{~cm}$
Let height of equilateral triangle $=h \mathrm{~cm}$
Using Pythagoras' on half of equilateral triangle:

$$
\begin{array}{ll}
x^{2}+h^{2}=(2 x)^{2} & \text { M1 } \\
x^{2}+h^{2}=4 x^{2} & \\
h^{2}=3 x^{2} & \\
h=x \sqrt{3} & \text { A1 }
\end{array}
$$

Perimeter of rectangle $=2(x+h)=2(x+x \sqrt{3})$
So,

$$
\begin{aligned}
& 2(x+x \sqrt{3})=20 \\
& x+x \sqrt{3}=10 \\
& x(1+\sqrt{3})=10 \\
& x=\frac{10}{1+\sqrt{3}}
\end{aligned}
$$

Perimeter of equilateral triangle $=2 x+2 x+2 x=6 x$

$$
=\frac{60}{1+\sqrt{3}}=21.961 \ldots
$$

$$
=22.0 \mathrm{~cm}(3 \mathrm{sf}) \quad \mathrm{A} 1
$$

$22 \quad O A$ and $A B$ are adjacent sides of a square and therefore perpendicular
Line through $A$ and $B$ has equation $y=-3 x+20$
Comparing with $y=m x+c$, gradient of $A B=-3 \quad$ M1
Gradient of $O A=\frac{-1}{-3}=\frac{1}{3}$
OA passes through origin so equation is $y=\frac{1}{3} x$
$A$ is intersect of lines so

$$
\begin{array}{ll}
\frac{1}{3} x=20-3 x & \text { M1 } \\
x=60-9 x & \\
10 x=60 & \text { A1 } \\
x=6 &
\end{array}
$$

When $x=6, y=\frac{1}{3} \times 6=2$ so $A$ is $(6,2)$
$O C$ will be $O A$ rotated $90^{\circ}$ so $C$ can be $(-2,6)$
[Other correct answer is $(2,-6)$ ]
Total 5

